UDK 336.763:519.865.5(497.5)"2018/2019"

USING GREY RELATIONAL ANALYSIS WITH FUZZY LOGIC IN PORTFOLIO SELECTION

Tihana Škrinjarić, Ph.D.
Assistant professor, Department of mathematics, Faculty of Economics and Business,
University of Zagreb, Croatia, tskrinjar@net.efzg.hr

Abstract

This research combines the Grey Systems Theory with the Fuzzy Logic in the process of selecting stocks into the portfolio. Since many financial data often include uncertainties with incomplete data, the mentioned approaches are quite useful for modelling such data. Based on the weekly data on 20 stocks on the Zagreb Stock Exchange for the period 2 January - 30 April 2019, the rankings from the Grey Relational Analysis are used to form membership functions within the Fuzzy Logic approach of making the final decision on investing. The dynamic analysis provides insights into the simulated portfolio characteristics which are compared to benchmark ones. The results of this study indicate that there exists potential in using the mentioned two approaches combined to achieve investment goals.

Keywords: *dynamic portfolio selection, Grey Incidence Analysis, fuzzy sets, portfolio evaluation, investment simulation*
JEL codes: *C61, G11, D81*

1. Introduction:

Although the portfolio selection problem is not new in the literature, new models, techniques and approaches which try to answer specific investment questions are constantly being developed daily. This is due to some of the shortfalls of existing approaches and methodologies, and due to ever-changing demands and preferences of investors, market conditions, etc. The parametric approach of modelling within the portfolio management theory and practice is probably the most common approach in the literature. It includes, in the majority of cases, time series econometrics, in which many models and methods impose assumptions that need to hold to use such an approach on empirical data. Several attempts of categorization of different models and techniques within this area of research have been done over the past couple of decades: Granger (1989), Taylor and Allen (1992), Ho et al. (2002), Wallis (2011). This was not an easy task to do, due to the fact that methodological advances, as well as computational ones (hardware and software), occur almost every day.

The area of portfolio selection includes many data, which is often incomplete, with many uncertainties. Thus, methodologies that deal with such data are not something new today. The Grey Systems Theory (GST) was developed just for such purposes when the decision-maker deals with many uncertainties within the process of comparing and modelling any type of system. Initial contributions are found in Deng (1982, 1989), after which this methodology spread out Asian literature in the majority of existing cases. Term "Grey", as often mentioned in the literature refers to grey data – uncertain data, on which we often have little true information, or the full range is not available. A historical overview of GST can be found in Liu et al. (2016) for those interested. The last couple of years have experienced a rise of interest in this theory, to apply it in many different aspects of the economy. This is due to many uncertainties that are present in an economy, and especially in financial markets. Some of the applications can be found in Škrinjarić and Šego (2019). On the other hand, the fuzzy set theory (FST) has been developing since the 1960s (Zadeh 1965), as deterministic models often failed to model real economic phenomenon. This is due to imperfect information, imprecise data and uncertainty. Thus, similar problems as in GST are tackled within the FST. Some applications within the portfolio selection can be found in Efranian et al. (2016) or Wang and Zhu (2002). Some of the advantages of fuzzy modelling within the portfolio selection include reduction of information loss which occurs in the traditional optimization model, linguistic constraints can be included in modelling and the possibility of obtaining a set of solutions instead of one (Fard and Ramezanzadeh, 2017). That is why some complementarities exist between the two approaches, the GST and FST. It is not surprising that research which includes both approaches within financial applications is growing in the last couple of years (Huang, Jane and Chang, 2008; Huang and Jane, 2009; Ma, Luo and Jiang, 2017).

By observing the previous literature, there exists a gap due to existing research not combining the GST and FST approach in which the stock rankings are built on. In particular, the final rankings based on the Grey Relational Coefficients (GRC) from the GST approach in this research are then used to construct membership functions within the FST approach. In that sense, the final decision on whether a stock will enter the portfolio or not will be based on defuzzified values of each criterion used to compare the stocks. Moreover, the criteria used in this research are based on the investor's utility theory and the portfolio return distribution moments; in particular, the first four moments due to their economic interpretation (see Athayde and Flores, 2004 or Jurczenko and Maillet, 2005). In that way, this research focuses on using uncertain data and applying the two approaches suitable for such data in order to obtain final crisp values which are comparable when making the decision on portfolio construction and rebalancing over time. The first part of the empirical analysis uses data on the first four moments of return distributions to calculate GRCs which are usually used to construct final values (Grey Relational Degree) which are usually compared and subsequently ranked. However, the second part of the analysis within the fuzzy set theory uses the GRCs to construct membership functions which give information on the probability that a stock's distribution moment belongs to a specific set or not. Based on the Sugeno (1985) model of deffuzyfing the output values, final investment decisions are made and simulated for a sample of 20 stocks on the Zagreb Stock Exchange. The procedure is repeated over a period of time to obtain insights into the usefulness of this approach for a dynamic portfolio rebalancing.

The rest of the research is structured as follows. The second section gives an overview of related previous research. The methodology is described in the third section, which includes description of GST and FST. The fourth section describes the data used in the study, with investment strategies simulated and compared one to another. The final section concludes the research.

2. Previous related research

The related literature with respect to methodologies used in this study and with respect to the area of applications is growing rapidly in the last couple of years. There are a lot of different models from the Grey Systems Theory which are applied within many different financial concepts; which is true for the fuzzy modelling approach as well. One group of papers utilizes models and approaches which are useful for comparison purposes when making decisions on whether to invest or not into specific assets. Other group tries to forecast future values of return, price or other relevant time series. Geographically, many different country stock markets have been analysed. When looking at the structure and the length of the research, many different approaches exist. Some authors published very short studies with some basic research results. Others include comparisons with different methodologies or simulate some behaviour of interest based on the estimation and/or calculation results. Thus, it was found that many different approaches of modelling exist within the existing studies, making the Grey methodology very flexible to combine with others.

Grey models and approaches can be found in the following papers. Yongzhong and Hongjuan (2005) is a very short study in which comparison is made between two Grey Model (GM) models for forecasting Shanghai Composite Index. Based on 200 observations, authors predict the stock index value with the GM and Verhlust GM model and compare the forecasts of one and several steps ahead by using average absolute percentage error statistic. The results show that the GM model is better for shorter term forecasts. This is a simple and a straightforward research, however, no portfolio strategies or other analysis were performed. Huang and Jane (2009) combined the moving average autoregressive exogenous (ARX) model with Grey systems theory and rough set (RS) theories in order to predict future prices on electronic stock data in the New Taiwan Economy database. For the period from the first quarter of 2003 to the fourth quarter of 2006, authors simulated trading based on the results of the mentioned methodologies. The ARX model was used to predict future price movements. Then, the Grey model was used to rank the stocks based on selected criteria (financial data) and the RS theory was used to make investment decisions. This approach resulted in extraordinary returns. Hwang, Lin and Chuang (2007) combined the DEA approach with the Grey situation decision model on the Taiwanese stock market in the period 2001-2004. Using the data on financial ratios, the authors utilized an additive DEA model to obtain efficiency scores based on different criteria. The scores were used within the Grey modelling part to predict future movements of the values of financial variables of interest. A Nash nonlinear Grey Bernoulli model was used in Doryab and Salehi (2017) to predict future prices on the Teheran Stock Exchange (period: 2005-2015). Authors have used several models of stock price prediction and compared the errors of prediction. The mentioned model produced the lowest errors which authors interpreted as being superior to other observed approaches. Škrinjarić and Šego (2019a) have utilized the Grey Incidence Analysis on a sample of 55 stocks on the Zagreb Stock Exchange. Since the authors observed both the stock market and financial ratios data, the analysis was made based on the fiscal year 2017. Thus, such analysis can be useful on a quarterly basis, since the financial statements are available on those frequencies. Authors have shown how to utilize the results in selecting the stocks based on investor's goals with respect to many different criteria. The same authors (Škrinjarić and Šego, 2019b) have utilized GM (1,1) and (2,1) models in order to compare their forecasting abilities to the usual econometric models (such as ARMA – autoregressive moving average) of stock price/return modelling. Since GM models are useful for short time spans analysis, authors observed the period 2 January – 12 June 2019. Out of sample forecasts and portfolio trading simulations have shown that the GM (1,1) model was the best performing in terms of investor's utility.

Another group of research focuses mostly on the fuzzy logic within the portfolio selection. Shipley (2009) suggested a proactive fuzzy set-based model in which decisions are based on finding the strength of each rule within the Fuzz Controller, updating the set rules and recommending new actions as the system is continuously looping. Rubell and Jessy (2015) developed an expert system for daily stock price prediction so that trading strategies could be simulated for the sample of 25 NASDAQ stocks in the period 2011-2015. Results of this research were promising, as the simulated obtained profits were better compared to other existing approaches which were compared to the one used in this study. Nakano, Takahashi and Takahashi (2017) were motivated by the previous success of fuzzy logic (FL) applications within the technical and fundamental analysis. Authors have combined state space models with FL on a sample of 8 stock market indices (Japanese, American, emerging markets FTSE index, etc.), by using monthly data for the period April 2003 to March 2016. Razi (2014) combined the Grey Relational Analysis (GRA) with Fuzzy Inference System (FIS) in order to select an optimal portfolio of investments regarding environmental investment. Several steps were conducted within this paper: firstly, the GRA was used to rank the potential projects based on different important criteria; secondly, FIS was used to predict the risk of the portfolio based on the observed projects. Thirdly, since the projects were dependent on the environmental factors, these factors were included in the multiple criteria selection process, and finally, based on the optimal Pareto solution, the best solution was chosen via optimization. Other possible combinations of complementary methodologies can be found in Lajevardi and Razi (2014). The possibilities of combining the methodologies are constantly growing. That is why the literature is rapidly expanding. However, the mentioned approaches do not take into account the Grey methodology results as fuzzy numbers as they will be observed in this research. Thus, we proceed to the calculations in order to obtain insights into the optimal portfolio structure based on the return distribution characteristics.

3. Methodology description

3.1. Grey Relational Analysis

The Grey Relational Analysis (GRA henceforward) is a methodology used within the area of grey numbers and systems where uncertainty is present when the decision has to be made. The basic idea of GRA is as follows (Liu and Lin 2006, 2010; Kuo et al. 2008). Decision-maker has to compare I alternatives by comparing their J behavioural sequences, $i \in \{1, 2, ..., I\}$, $j \in \{1, 2, ..., J\}$. I alternatives in this study refer to stocks which investor compares, and the J denotes the number of criteria the investor makes comparisons. The data can be summarized in a matrix in the following form:

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$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1J} \\ x_{12} & x_{22} & & x_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ x_{I1} & x_{I2} & \cdots & x_{IJ} \end{bmatrix}, \tag{1}$$

where x_{ij} denotes the behavioural value i for the j-th criterion, i.e. $(x_{i1}, x_{i2}, \dots, x_{iJ})$ is the behavioural sequence for the alternative (stock) i. Matrix X has to be normalized so that the data can be compared with respect to criterion j (see Huang and Liao 2003). Thus, each row in matrix X is normalized, with respect to the criterion which is used to compare the alternatives. If the observed sequences have to be the largest possible, sequences $(x_{i1}, x_{i2}, \dots, x_{iJ})$ are normalized by using the formula:

$$y_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} . \tag{2}$$

Normalization for sequences where data should be minimal possible is done via formula (3):

$$y_{ij} = \frac{\max_i x_{ij} - x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} , \tag{3}$$

and if there exists a predetermined optimal desired value x^*_j which should be achieved, then the sequences are compared to that optimal value and the normalization is done as follows:

$$y_{ij} = \frac{\left| x_{ij} - x_j^* \right|}{\max_i x_{ij} - x_j^*} . \tag{4}$$

The data in the new matrix *Y* which consists of normalized values is now within the range [0,1], with the value being closer to unit value being better. The next step consists of comparing the differences, i.e. the distances of each normalized value to a referent sequence y^* with respect to each criterion *j*. We follow Kuo et al. (2008), in which the unit value is the best reference for every criterion, thus, each distance is calculated as $\Delta y_{ij} = \left| y_{ij} - 1 \right|$.

The Grey Relational Coefficient (GRC) for every alternative *i* and its behavioural sequence *j* is calculated as the ratio $G_{ij} = \frac{\Delta_{\min,j} + p\Delta_{\max,j}}{\Delta y_{ij} + p\Delta_{\min,j}}$, where *p* is called distinguishing coefficient, with $p \in [0,1]$, and $\Delta_{\min,j} = \min\left\{\Delta y_{1,j}, ..., \Delta y_{I,j}\right\} \ \forall j$, $\Delta_{\max,j} = \max\left\{\Delta y_{1,j}, ..., \Delta y_{I,j}\right\} \ \forall j$. Coefficient *p* is used to expand or compress values G_{ij}, it does not affect the rankings of every alternative within the sequence *j*. Finally, the Grey Relational Degree (GRD) for every alternative (stock in this research) is calculated as a weighted average of Grey Relational Coefficients as follows:

$$GRD_i = \sum_{j=1}^{J} w_j G_{ij} . \tag{5}$$

w_j is the weight of criterion *j*, where $\sum_{j=1}^{J} w_j = 1$ holds. Values in (5) are the degrees of similarities between every alternative *i* and the referent sequence y^* (i.e. unit values), where the greater the value of GRD_i in (5) is, the closer the alternative is to the best sequence. Rankings of every alternative are then made from the greatest to the lowest values. The problem here is how to choose weights in (5). These could be made based on the researcher's experience, investor's preferences towards criteria he is using in his decision making process, etc. for details, please refer to Liu et al. (2016).

3.2. Fuzzy logic

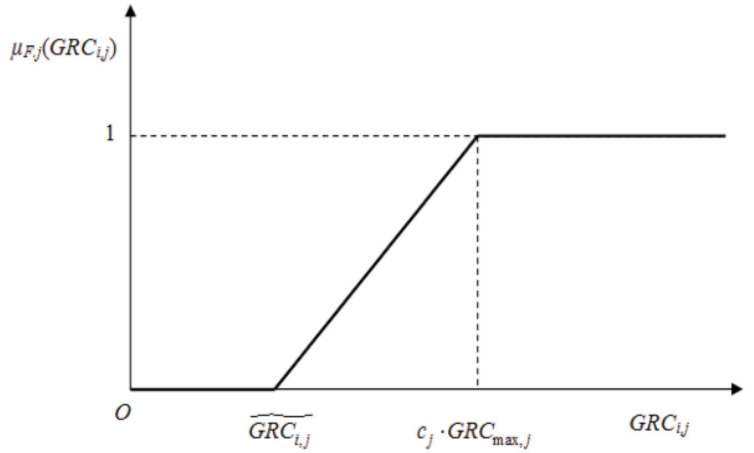
Fuzzy logic and fuzzy set theory are based on uncertainty and partial truth, when decisions are made based on imprecise information, which is often non-numerical (for introduction and details, please refer to Jang et al. 1997; and Zimmermann 2001). The main difference between the crisp set and the fuzzy set theory is that in the crisp set theory, an element is either a member of the set or not. On the other side, the fuzzy set theory includes a probability function of belonging to a set or not. Idea is to fuzzify the input values regarding the membership functions, define the rules to compute fuzzy output functions and execute them, and finally, to de-fuzzify the fuzzy output into crisp output values. There are many ways and approaches to fuzzifying the input values and de-fuzzifying the output, with different applications in finance (Li and Xu 2013). The crisp set can be written as follows: $C = \{x \mid x \in X\}$, whereas a fuzzy set has notation $F = \{(x, \mu_F(x)) \mid x \in X\}$, where $\mu_F(x)$ is the membership function of x in set F that maps X to the membership space.

The basic usage of the fuzzy set theory in this research is based on calculating the GRCs in Grey methodology (see the previous subsection). Instead of calculating the GRDs via formula (5), membership functions will be formed for every GRC so that fuzzy sets are defined for every criterion j. Thus, the membership function for very GRC is given with formula (6):

$$\mu_{F_j}(GRC_{i,j}) = \begin{cases} 0, & GRC_{i,j} < GRC_{i,j} \\ 1 + \dfrac{GRC_{i,j} - c_j \cdot GRC_{\max,j}}{c_j \cdot GRC_{\max,j} - GRC_{i,j}}, & GRC_{i,j} \leq GRC_{i,j} < c_j \cdot GRC_{\max,j} \\ 1, & GRC_{i,j} \geq c_j \cdot GRC_{\max,j} \end{cases} \quad (6).$$

$\mu_{F_j}(GRC_{i,j})$ is a membership function with respect to criterion j, $GRC_{i,j}$ is the value for alternative i ranked based on criterion j from the Grey methodology, $GRC_{i,j}$ is a threshold value. If the $GRC_{i,j}$ value is below the mentioned threshold, value of the membership function should be equal to 0 and the element does not belong to the set. If the value $GRC_{i,j}$ exceeds the threshold value, but is below some other threshold value $c_j \cdot GRC_{\max,j}$, where c_j is the percentage referring to the maximal value the Grey Relational Coefficient can achieve with respect to the criterion j, then the membership function is the probability $1 + \dfrac{GRC_{i,j} - c_j \cdot GRC_{\max,j}}{c_j \cdot GRC_{\max,j} - GRC_{i,j}}$ (a line function in essence). Finally, if the individual value $GRC_{i,j}$ exceeds the value $c_j \cdot GRC_{\max,j}$, then the probability of belonging to the set is equal to 1. Graphically, the function (6) is shown in Figure 1, where the full broken line is the value of the function (6).

Figure 1. Membership function defined in (6)



Source: author

The threshold value $GRC_{i,j}$ and c_j should be chosen based on the topic of the research. In this paper, since the investor has to make decisions based on return distribution moments, values of $GRC_{i,j}$ and c_j will be chosen so that in every week there is always at least one stock in the portfolio. In order to defuzzify the output values, the Sugeno (1985) model was used to define the crisp output values. The model is simple and straightforward, where the final rankings are made as follows, based on the first order Sugeno model. If two fuzzy inputs are used, then the fuzzy rule is: *if x is F_1 and y is F_2 then z = f(x,y)*, where F_1 and F_2 are fuzzy sets and *z* is the crisp function. The order of the polynomial for *z* can be chosen by the researcher. Thus, we choose the first-order (linear function) for every criterion *j*, so that the function is as simple as possible. Finally, two rules are defined, again, due to keeping the decisions simpler. The first rule implies that if all $GRC_{i,j}$ values are large, then the *z* function is the sum of all *GRS*-s. The opposite is true for small values of $GRC_{i,j}$; the negative values are summarized into *z*. The output is finally calculated as the ratio $Z = \sum_{j=1}^{J} \lambda_j z_j \ / \sum_{j=1}^{J} \lambda_j$, where each λ_j is the degree of truthfulness of the premise of the *j*-th rule. The ratios are compared for every stock and the ranking is made from largest to smallest values. The larger the value of *Z* for a stock is, the better its ranking is. As a simple procedure, we choose those stocks which have values of *Z* above the average value for a given time period the calculation is based on.

4. Empirical results

4.1. Data description

The empirical analysis of combining the Grey methodology with fuzzy logic is based on a sample of 20 stocks quoted on the Zagreb Stock Exchange. Daily data on stock prices have been collected from Investing (2019) and ranges from 2 January 2018 until the end of April 2019. Based on the prices, the return series have been calculated for every day. In the last step, average weekly returns have been calculated for every working week in the time sample.

The main criterion for choosing the stocks in the analysis was their liquidity in the observed period. Detailed information on the names and sector classification of each stock is given in Table A1 in the Appendix. Table 1 depicts the basic descriptive statistics of the returns of each stock used in the study. As can be seen, stocks have different characteristics of distribution moments. However, these are just the averages of the entire observed period.

Table 1. Descriptive statistics for weekly return series

Stock / Characteristic	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	JB	Probability
ADGR	0.000	0.000	0.010	-0.013	0.004	-0.349	4.253	6.005	0.050
ADPL	0.000	0.000	0.011	-0.008	0.004	0.380	3.295	1.942	0.379
AREN	0.000	0.000	0.012	-0.009	0.004	0.410	3.405	2.440	0.295
ATGR	0.001	0.002	0.017	-0.021	0.007	-0.849	5.468	26.171	0.000
ATPL	-0.002	-0.003	0.069	-0.032	0.013	2.290	14.940	477.005	0.000
DDJH	0.001	-0.003	0.054	-0.040	0.017	0.635	3.583	5.696	0.058
DLKV	-0.003	-0.003	0.038	-0.040	0.012	0.259	5.070	13.282	0.001
ERNT	-0.001	0.000	0.020	-0.028	0.007	-0.958	7.572	71.662	0.000
HT	0.000	0.000	0.008	-0.011	0.003	-0.340	4.774	10.525	0.005
INGR	0.003	0.000	0.075	-0.022	0.013	2.318	13.713	397.412	0.000
KONL	-0.001	0.000	0.014	-0.033	0.007	-1.226	7.179	68.453	0.000
KRAR	0.000	0.000	0.034	-0.019	0.009	1.041	6.154	41.651	0.000
LKPC	0.005	0.000	0.106	-0.071	0.030	1.541	6.445	62.335	0.000
LUKA	0.001	0.000	0.048	-0.029	0.013	1.383	6.897	66.618	0.000
PODR	0.001	0.000	0.014	-0.010	0.004	0.524	4.503	9.794	0.007
RIVP	0.000	-0.001	0.032	-0.016	0.007	1.910	10.780	219.077	0.000
TPNR	-0.003	0.000	0.052	-0.038	0.017	1.177	6.055	43.395	0.000
ULPL	0.001	-0.002	0.078	-0.056	0.026	0.676	4.024	8.393	0.015
VART	0.007	0.000	0.085	-0.036	0.023	0.981	3.964	13.928	0.001
ZBB	0.000	0.000	0.024	-0.039	0.008	-1.129	10.895	196.644	0.000

Source: author's calculation.

Note: Std. Dev. denotes standard deviation; JB denotes Jarque-Bera test value for the null hypothesis of normal distribution, Probability is the p-value of the JB test.

Since this paper compares dynamic strategies over the entire period, we calculate for every week and every stock the return, standard deviation, skewness, and kurtosis. The four distribution moments will be used as the criterion to compare stocks every week (thus, $J = 4$, see the methodology section). The main rationale for why we use the four distribution moments is based on the previous theoretical and empirical work. Firstly, the theory on the investor's utility function depending on the first four moments of the return and portfolio distribution has been developed for several decades now. First interest towards higher distribution moments with respect to the stock and portfolio return series can be found in Kendall and Hill (1953), Mandelbrot (1963a, b), Cootner (1964), Fama (1965), etc. Arditty (1967) has shown that individual investor is more prone towards greater asymmetry of portfolio return, which is in accordance with decreasing absolute risk aversion. Alderfer and Bierman (1970) shown that investors are willing to suffer lower mean returns for a given return distribution if this meant that the asymmetry of such distribution will be greater (a trade-off was found). Graddy and Homaifar (1988) extended the portfolio selection model of Markowitz (1952, 1959) to the first four moments; whereas newer theoretical developments utilize Taylor's expansion of the utility function for the first *m* moments[26] of the distribution, such as Jondeau and Rockinger (2003), Athayde and Flores (2004) or Jurczenko and Maillet (2005).

Secondly, such theoretical findings were utilized in the empirical research, in which authors use the first four moments of the return in related research (Škrinjarić and Šego 2019). Thirdly, the data used in this study is not normally distributed, as can be seen in Table 1. Expect for stocks ADPL, AREN and DDJH, all of the distributions were found to be non-normal. This contributes to the decision on using higher moments of return distributions when comparing the stocks.

4.2. Strategies description

For the comparison purposes, several benchmark strategies have been simulated so that the results from the approach in this study can be compared to the approaches which do not utilize such methodologies. The benchmark strategies include the following ones. The equal weights strategy is the simplest one in which all stocks have equal weights in the portfolio throughout the whole observed period. This means that each stock makes 1/20 of the entire portfolio structure. Thus, this strategy is a passive one, as the investor does not observe the changes in the market and he does not rebalance the portfolio over time. The second benchmark strategy is the random weights one. In each week, we give random weights to all of the 20 stocks. This excludes any possible subjectivities within the decision-making process. The third strategy is called the minimum variance strategy. This one is based on minimizing the portfolio risk within the Markowitz (1952) model, where the following problem is solved in each week:

$$\begin{aligned} \min_{x_t} \quad & \boldsymbol{x}_t' \Sigma_t \boldsymbol{x}_t \\ \text{s.t.} \quad & \boldsymbol{e}' \boldsymbol{x}_t = 1 \quad , \\ & \boldsymbol{x}_t \geq \boldsymbol{0} \ \forall t \end{aligned}$$

where $\boldsymbol{x}_t$ is the vector of stock weights in week t, Σ_t is the variance-covariance matrix in each week t, and $\boldsymbol{e}$ is the $(1 \cdot n)$ vector of unit values, and n is the number of stocks which enter the portfolio.

The main strategies which are based on the GST and FST are the following ones. In each strategy, the input values for the fuzzy logic part of the calculation are taken from the first step in which the GRCs were calculated for every return distribution moment. Then, by varying the values of the threshold value $GRC_{i,j}$ and c_j in (6) were changed so that we can obtain different strategies based on possible thresholds which investor is interested in. Based on the function (6) and the defined function z and ratio Z (please see section 3.2), we obtained crisp

[26] With the emphasis on the first four moments.

values for every distribution moment for every stock in week *t*. All of those stocks for which we obtained positive results regarding investing in them in week *t* are given equal weights in the portfolio.

The threshold value $GRC_{i,j}$ was chosen to be the average GRC value between all of the stocks in the respective week *t*. in that way, every week we have stocks present in the portfolio. If this value was set to be higher, the result could lead to none or 1 stock in the portfolio, which would not be favourable for the diversification possibilities. The value of c_j was then chosen to be 0.5; 0.6 and 0.7. Thus, the first three portfolios are based on the average value of GRCs for the threshold value, but the value of c_j differ. The greater this value is, the "better" stock will enter the portfolio (in terms of better GRC values). These first three portfolios are called "fuzzy 0.5", "fuzzy 0.6" and "fuzzy 0.7". Furthermore, we wanted to obtain additional strategies in which we wanted to increase the threshold value $GRC_{i,j}$, so that even better stocks enter the portfolio. Instead of using the average value of GRCs, we multiply the average value by 1.2 so that the lowest threshold is now a greater value, which will exclude some stocks which previously entered the portfolio. To strategies "fuzzy 0.6" and "fuzzy 0.7" we add the "+1,2 mean" which indicates this inclusion.

Thus, in total, we observe 3 benchmark strategies: "equal weights" (or "average"), "random" and "minimum variance" portfolios; and the strategies based on Grey approach combined with Fuzzy Logic include: "fuzzy 0.5", "fuzzy 0.6", "fuzzy 0.7", "fuzzy 0.6 + 1,2 mean", "fuzzy 0.7 + 1,2 mean". Every week, the portfolios are rebalanced. This is either based assigning the weights at random ("random" strategy); by solving the problem of minimising the portfolio variance ("minimum variance" strategy) or by including those stocks which should enter the portfolio based on the Grey and FL approach (5 lastly mentioned strategies). Summarized, in each week the investor evaluates the stocks based on the defined criteria within each approach and buys the best evaluated stocks in the next one. The procedure is repeated until the end of the observed period. With the exception of the "equal weights" strategy, in all other strategies, we include transaction costs equal to 1% of the total transaction. The results are presented in the following subsections.

4.3. Main results

In each week we calculate the portfolio return and risk series based on the return and risk series of every stock which enters the portfolio. The cumulated values of each portfolio are shown in Figure 1, where each portfolio started with a unit value. It can be seen that the best performing portfolios in terms of the portfolio value over time are "fuzzy 0.6 + 1,2 mean" and "fuzzy 0.7 + 1,2 mean". The majority of the time, the two portfolio values overlap. This is due to a similar portfolio structure of the two mentioned strategies. Furthermore, it is seen that at the end of the observed period, both portfolios have values greater than 3 monetary units. This means that if the investor has invested one monetary unit in such portfolios at the beginning of the observed period, he would earn more than 2 monetary units at the end. All other Grey with Fuzzy Logic strategies earn money as well: a little over 0.8 monetary units. The benchmark strategies performed the worst, with some portfolio losses. Such substantive results with respect to the used methodologies in this research are in line with previous literature.

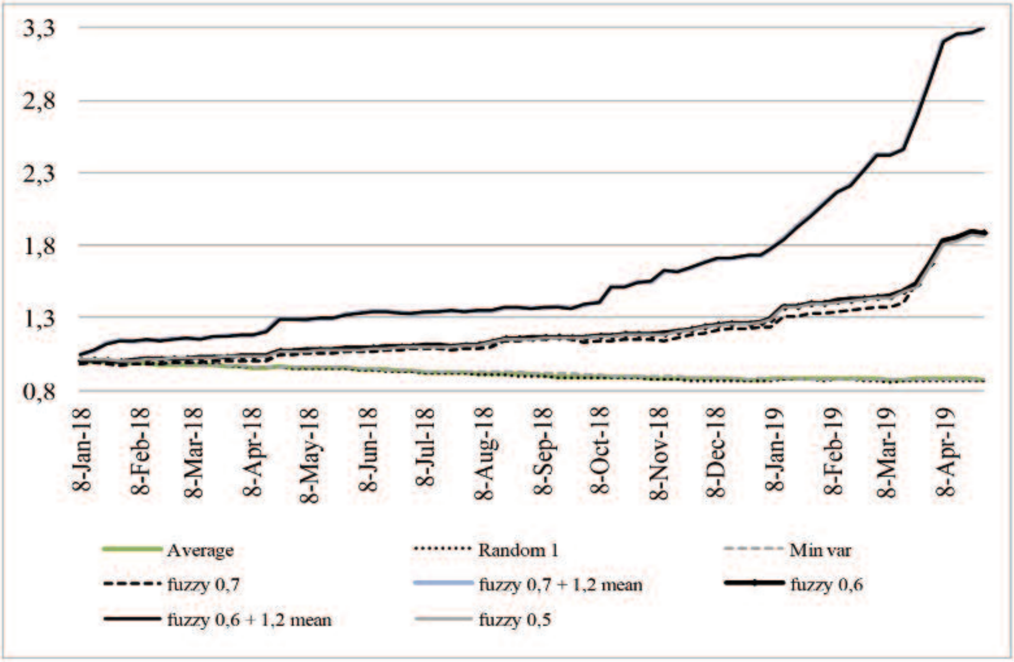


Figure 1. Cumulative portfolio values of simulated strategies

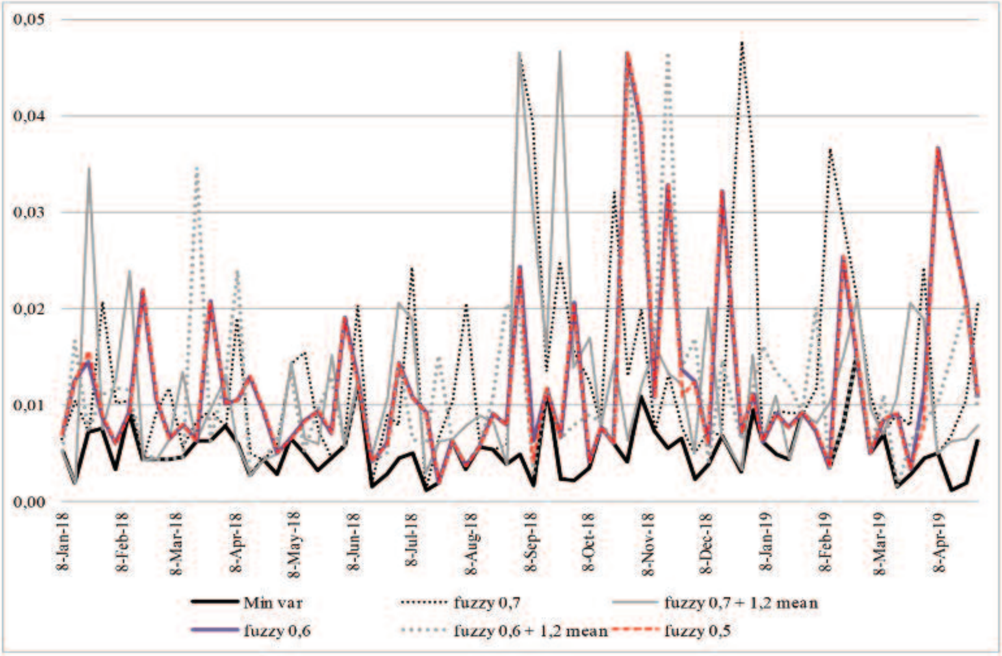


Figure 2. Portfolio risks of simulated strategies

However, investors are interested in the risk series as well. Thus, portfolio variances of each strategy based on the Grey and FL approaches have been calculated and compared to the minimum variance portfolio. This benchmark portfolio was chosen as this approach within the Markowitz portfolio framework ensures the minimal risk series. Figure 2 depicts the risks series, where it is obvious that in the majority of the time, the minimum variance portfolio does achieve minimal portfolio risk. Other strategies have greater risks series the majority of the time, especially during the second part of 2018 and in 2019. This is not something unexpected, as the Grey and FL portfolios obtained great returns in that period (Figure 1), which was followed with greater risks.

Figure 3 shows the dynamics of the cumulative portfolio values per one unit value of portfolio risk. In that way, more insights are obtained into both measures important for the investor at once. The previously mentioned best ranked portfolios in Figure 1, "fuzzy 0.6 + 1,2 mean" and "fuzzy 0.7 + 1,2 mean", are best once again. Although these portfolios had greater portfolio risks compared to the minimum variance portfolio, the returns investor could have realized were of such values that even when adjusting the return series with risks, both strategies obtained substantial results.

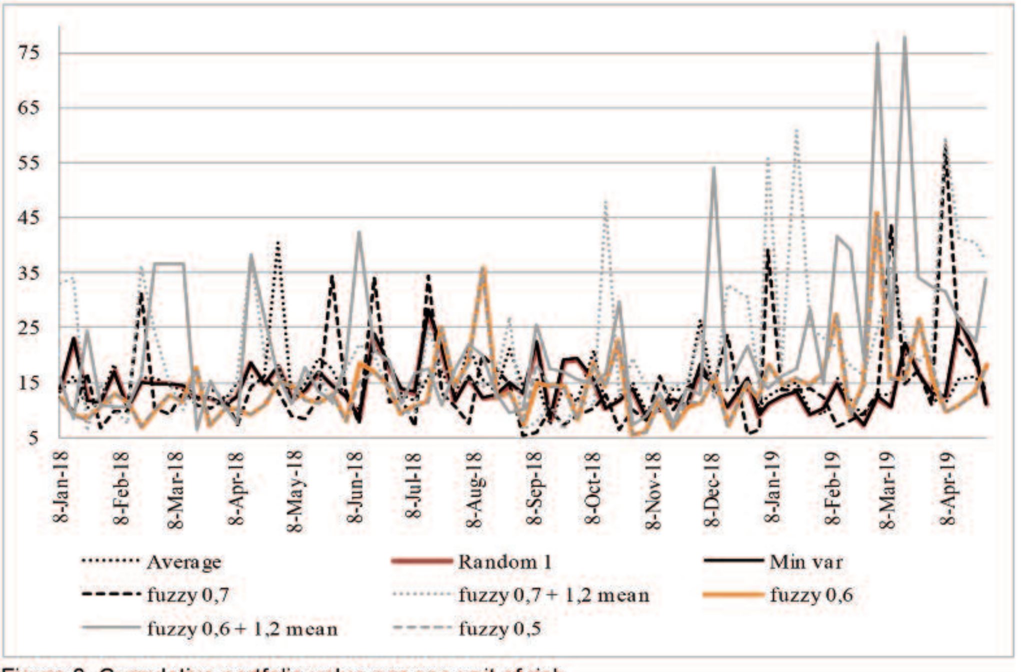


Figure 3. Cumulative portfolio value per one unit of risk

Finally, Table 2 gives details on return and risk series, with the estimation of CE (Certainty Equivalent), which is the value of the utility an investor would achieve by playing an uncertain gamble. The CE is estimated as follows: $CE \approx E(R_p)\text{-}0.5\gamma\sigma^2$, where $E(R_p)$ is the expected return of a portfolio, γ is the coefficient of absolute risk aversion and the variance represents the portfolio variance, i.e. risk. CE was estimated for every week for all of the observed strategies for γ = 1, 5 and 10 (values usually range from 1 to 20, as Guidolin and Timmerman, 2005 have shown that the CE is robust within this range). Since all of the series observed in Table 2 are time series, we calculated the mean, median, minimal and maximal values of each of the portfolio characteristics. The "fuzzy 0.7 + 1,2 mean" strategy is the best one with respect to the return series. This was expected, due to Figure 1. This strategy provided the best return series throughout the entire observed time span. However, all of the strategies realized some losses as well, as the minimal return values were actually losses. The minimum variance strategy at one point ensured the minimal loss (-0.007), however, the strategies based on Grey and FL combination were not very far away (-0.008 for “fuzzy 0.5” and “fuzzy 0.6”).

The lowest risk was obtained via the minimum variance portfolio, but it was followed with all other strategies with only the exception of the equal weights strategy. The CE values, which estimate the utility from such strategies, which is based on the return and risks series at the same time shows that the more the investor is risk averse (greater values of γ), he should aim towards some of the Grey and FL strategies. This is due to their return series being exceptional which compensated the greater weights added to the portfolio risk.

Table 2. Portfolio characteristics of simulated strategies

Strategy		Equal weights	Minimum variance	Random	Fuzzy 0.5	Fuzzy 0.6	Fuzzy 0.6 + 1.2 mean	Fuzzy 0.7	Fuzzy 0.7 + 1.2 mean
Return	Mean	-0.002	-0.002	-0.002	0.009	0.009	0.017	0.009	**0.017**
	Median	-0.002	-0.002	-0.002	0.005	0.005	0.009	0.004	**0.009**
	Maximum	0.007	0.004	0.008	0.088	0.088	0.088	0.088	**0.088**
	Minimum	-0.013	**-0.007**	-0.015	-0.008	-0.008	-0.009	-0.017	-0.009
Risk	Mean	**0.0049**	0.0052	0.0052	0.0115	0.0116	0.0105	0.0126	0.0107
	Median	**0.0043**	0.0049	0.0049	0.0089	0.0090	0.0082	0.0094	0.0082
	Maximum	0.0151	**0.0151**	0.0151	0.0466	0.0466	0.0467	0.0478	0.0467
	Minimum	0.0006	**0.0011**	**0.0011**	**0.0011**	**0.0011**	**0.0011**	**0.0011**	**0.0011**
CE γ=1	Mean	-0.036	-0.037	-0.037	-0.041	-0.041	**-0.028**	-0.041	-0.029
	Median	-0.035	-0.036	-0.036	-0.041	-0.041	**-0.034**	-0.044	-0.035
	Maximum	-0.011	-0.014	-0.012	0.044	0.034	0.049	**0.088**	0.074
	Minimum	-0.071	**-0.060**	-0.071	-0.108	-0.108	-0.113	-0.109	-0.116
CE γ=2	Mean	**-0.070**	-0.072	-0.072	-0.095	-0.095	-0.084	-0.099	-0.084
	Median	**-0.069**	-0.071	-0.070	-0.088	-0.088	-0.079	-0.093	-0.078
	Maximum	-0.026	-0.031	-0.031	-0.001	-0.020	0.009	**0.018**	**0.018**
	Minimum	-0.133	**-0.121**	-0.132	-0.216	-0.216	-0.221	-0.219	-0.224
CE γ=10	Mean	-4.934	-4.932	-4.932	-4.887	-4.886	**-4.881**	-4.883	-4.881
	Median	-4.937	-4.933	-4.932	-4.896	-4.896	**-4.883**	-4.892	-4.889
	Maximum	-4.887	-4.876	-4.887	-4.720	**-4.720**	-4.762	-4.756	-4.768
	Minimum	-4.979	-4.970	-4.970	-4.961	-4.963	-4.945	-4.967	**-4.942**

Note: CE denotes Certainty Equivalent. Bolded values indicate best performance in each row.

5. Conclusion

This research belongs to the group of papers which utilize methods of the uncertain decision-making process when data is scarce, uncertain and/or incomplete. The main focus was made on the decision making the process of the structure of the portfolio based on the first four moments of the return distribution of stocks which are being compared. One of the two approaches of the study includes the Grey Relational Analysis, which is a relatively unknown methodology in the literature when compared to other popular approaches of stock price and risk modelling. Since it has straightforward interpretation, previous research, as well as this one, finds that it can be very useful in the portfolio selection process; either by itself or by combining it with other complementary approaches. The second approach used in this study is the Fuzzy Logic, which is also developed to aid the decision-making process with grey data. The fuzzy set theory is more known in the literature when compared to the GST. However, financial applications are rising mostly in the last decade. This is due to greater data availability and computational possibilities of computers.

Based on the results, several things could be said. Firstly, the analysis showed that by combining the Grey methodology with Fuzzy Logic could provide increases in portfolio value over time. This is in accordance with previous literature. Secondly, the risks which have to be endured for greater return series, although greater than the risks from usual optimization of the portfolio structure (e.g. minimal variance portfolio), are still not sufficiently large to diminish or cancel the return series. Thirdly, the greater the values of the thresholds chosen in the membership function within the fuzzy decision making are, the better stocks enter the portfolio. Term "better" refers to stocks that have greater first and third moments of the return distribution, and lower second and fourth moments. However, the threshold values have to be chosen based on the actual values within the problem itself, so that some stocks will enter the portfolio every week (so that the portfolio is not empty, or consisting of only several stocks), but not all of them (then, there is no point in using such analysis).

The approach of this research has shown that some investment goals could be achieved by utilizing a combination of two approaches which are useful when data is uncertain and incomplete. This is especially true for the return series. The risk series should be in focus in future research though, as best performances were obtained in terms of portfolio value over time, even when accounting for the transaction costs. The future work should focus on some of the following issues: finding best trade-off between the mentioned threshold values and the number of stocks within a portfolio; trying to give weights in the Grey or FL part of the analysis which will emphasize specific investor goals with respect to his preferences and/or possibilities (constraints); defining other membership functions within the FL part of the analysis, in order to obtain even better results with respect to any of the return distribution moments; etc. Thus, a lot of work is still left for future research to deal with.

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Appendix

Table A1. Stock abbreviations with full names

Abbreviation	Full name	Sector
ADGR	Adris Group d.d. for managing and investing	Management activities
ADPL	Ad Plastik d.d. for motor vehicles parts production and plastic mass products	Other parts and motor vehicles parts
AREN	Arena Hospitality Group d.d. for tourism and hospitality	Hotels and similar accommodation
ATGR	Atlantic Group d.d. for domestic and foreign trade	Non specialized wholesale
ATPL	Atlantska plovidba d.d. group for international transport of people and goods	Maritime and coastal transport
DDJH	Đuro Đaković group d.d.	Management activities
DLKV	Dalekovod d.d. for engineering, production and construction	Construction of electrical power lines and telecommunication
ERNT	Ericsson Nikola Tesla d.d. for telecommunication system and device production	Telecommunication equipment production
HT	Hrvatski Telekom d.d.	Wired telecommunications activities
INGR	Ingra d.d. for construction of investment objects, imports, exports and dealerships	Engineering and related technical consultation
KONL	Končar electroindustry d.d.	Electric motors, generators and transformers production
KRAR	Kraš food industry d.d.	Cocoa, chocolate and confectionery products production
LKPC	Luka Ploče d.d. for maritime traffic services, port services, goods storage and shipping	Cargo handling
LUKA	Luka Rijeka d.d. for maritime traffic services, port services, goods storage and shipping	Cargo handling
PODR	Podravka food industry d.d.	Other food processing, fruit and vegetables conservation
RIVP	Valamar Riviera d.d. for tourism	Hotels and similar accommodation
TPNR	Tankerska Next Generation shipping d.d.	Maritime and coastal transport
ULPL	Uljanik Plovidba d.d. Maritime transport	Maritime and coastal transport
VART	Varteks d.d. Varaždin textile industry	Production of other clothes
ZBB	Zagreb bank d.d.	Other money intermediation

Source: Investing (2019) and Zagreb Stock Exchange (2019). Note: d.d. denotes joint stock company